ExB drift instability, introduction, physics and benchmark definitions

Jean-Pierre BOEUF
LAPLACE, CNRS, University of Toulouse

OUTLINE

1. **The LANDMARK project**

2. **EXB instabilities – PIC simulation benchmarks – Physics**
   - Introduction on ExB Electron Drift Instability – Ion Acoustic instability
   - 1D azimuthal , 2D radial-azimuthal , and 2D axial-azimuthal PIC simulations and benchmaks
   - Some remarks on the physics

3. **Conclusion**

LANDMARK project – Context and Objectives

Low temperAture magNetizeD plasMA benchmaRKs

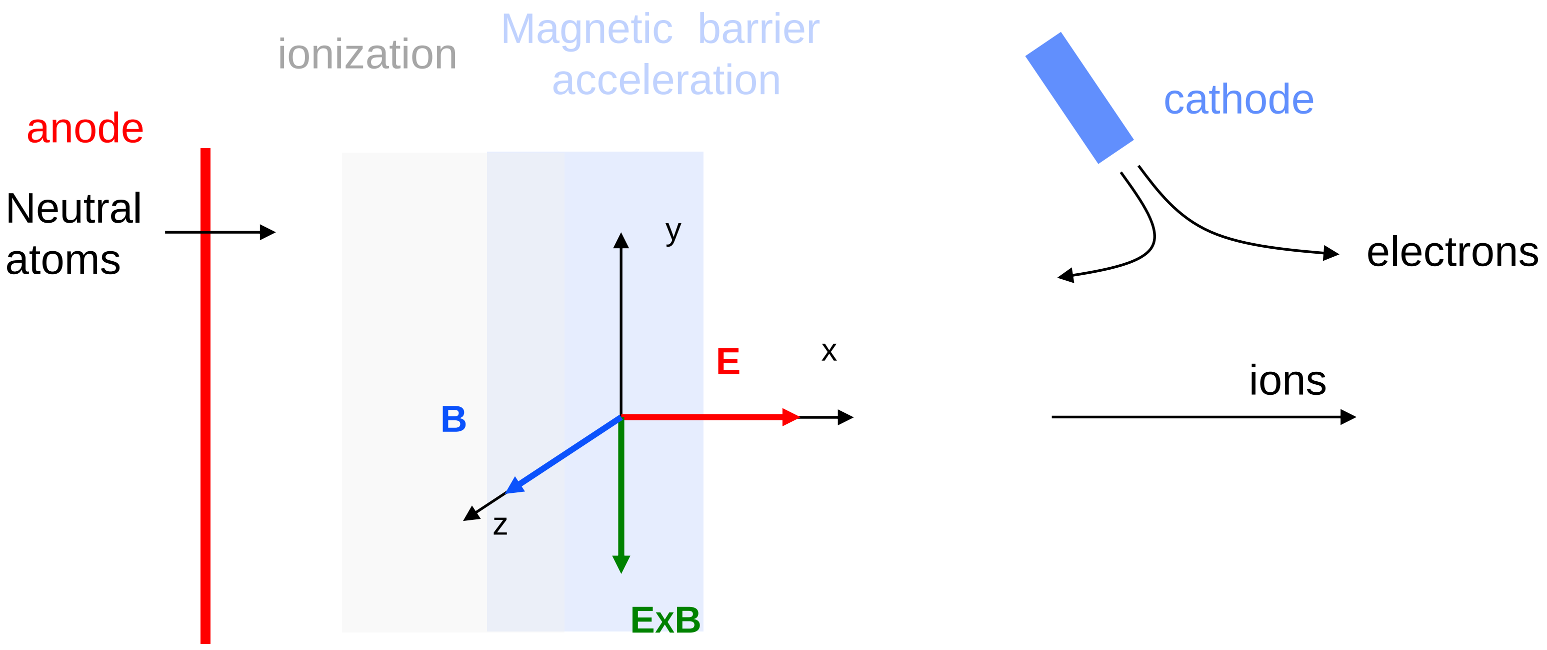
- Need to organize the community to get a better understanding of (non-fusion) ExB plasmas (where electrons are strongly magnetized while ions are not)

- The **LANDMARK** project aims at:
  - Providing an open forum for evaluating methods of description of plasma transport in non-fusion magnetized plasmas
  - **Defining benchmark** test cases for PIC, fluid and hybrid models of magnetized plasmas
  - ***Addressing physics issues*** related to anomalous transport across magnetic field: instabilities, plasma wall interactions and their influence on particle and energy transport
  - Facilitating international collaboration and mutual understanding among researchers – Publishing benchmark results

https://www.landmark-plasma.com

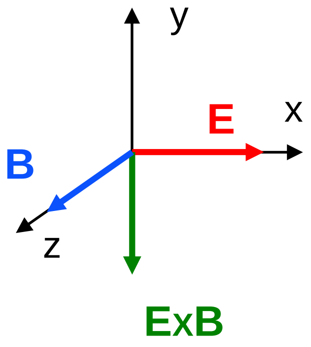
Anomalous transport in ExB configurations

- **ExB configuration typical of Hall ion sources**



- Electrons are magnetized - Ions are not magnetized
- Closed drift in azimuthal ExB direction
- Instabilities in ExB direction due to large electron drift:
- Electron Cyclotron Drift Instability or ExB Electron Drift Instability

Anomalous transport in ExB configurations



- **Theory of Electron Cyclotron Drift Instability**
  or ExB Electron Drift Instability (ExB EDI)

  - 2D dispersion relation (k_z=0)

    When k_z=0 (direction // B), instabilities in small intervals in k_y , around multiples of the inverse of the Larmor radius. Large resonances at:

$$k_{y,n} = n \frac{\Omega_{ce}}{V_E} \qquad V_E = V_d = E / B$$

  - Finite k_z
    - For non-zero k_z or when non-linear effects are present (resonant broadening) the discrete nature disappears and the dispersion relation simplifies to a **modified ion-acoustic** type relation
    - Is there a transition to ion acoustic instability in conditions of Hall thrusters ?

Anomalous transport in ExB configurations

- **Transition from ExB EDI to ion acoustic instability**
  - Wave vector at maximum growth rate $k_{y,\max} \approx \frac{1}{\sqrt{2}\lambda_{De}}$ $\lambda_{y,\max} \approx 2\pi\sqrt{2}\lambda_{De}$
  - Angular frequency at max growth rate $\omega_{R,\max} \approx \frac{\omega_{pi}}{\sqrt{3}}$
  - Amplitude of field oscillations

    obtained by assuming that saturation due to ion-wave trapping $\left|\delta E\right| = \frac{1}{3\sqrt{2}} \frac{T_e}{\lambda_{De}}$

- **Wavelength for ExB EDI and ion acoustic instability**

ExB EDI $$\boxed{\lambda_{w,E\times B\ \ EDI} = \frac{2\pi}{k_y} = 2\pi\frac{V_E}{\Omega_{ce}}}$$

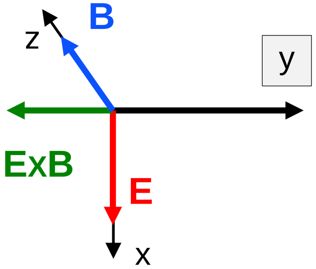
Ion Acoustic $$\boxed{\lambda_{w,IAI} = \frac{2\pi}{k_y} = 2\pi\sqrt{2}\lambda_{De}}$$

For E=200 V/cm, B=200 Gauss, ne~1017 m^{-3}, Te~50 eV

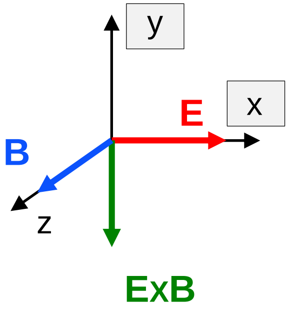
$\lambda_{w,EXB\ EDI}$ and $\lambda_{w,IAI}$ are both in the 1 mm range

LANDMARK project – PIC simulation Test Cases – Definitions

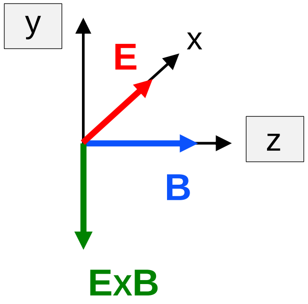
- **Test Case 1 - 1D azimuthal PIC simulation of the ExB EDI**
  - Constant and imposed axial Ex and radial Bz. Constant number of particles. Only azimuthal (ExB) direction is described by the model. Periodic boundary conditions. Finite length of acceleration region can be considered (i.e. re-injection of particles)



- **Test Case 2a - 2D axial-azimuthal PIC simulation of the ExB EDI**
  - Given axial length and periodic azimuthal length. Given axial distribution of radial magnetic field. Given ionization rate profile. No collisions. Given applied voltage. Radial direction not described

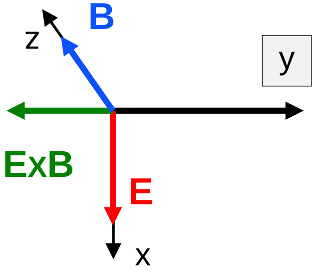


- **Test Case 2b - 2D radial-azimuthal PIC simulation of the ExB EDI**
  - Constant and imposed axial Ex and radial Bz, as in Test Case 1, but radial direction is described

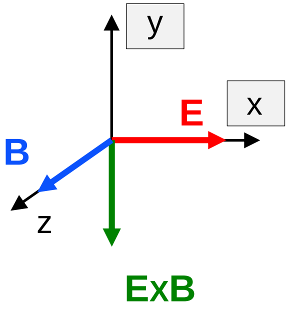


LANDMARK project – PIC simulation Test Cases – Issues

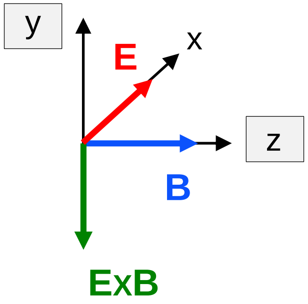
- **Test Case 1 - 1D azimuthal PIC simulation of the ExB EDI**
  - Study the development of the EXB EDI in relation with the dispersion relation. Evolution toward Ion Acoustic instability ? Effective collision frequency ? Role of periodic azimuthal length ? Role of finite length in axial direction (i.e. re-injection of particles). Role of numerical noise ?



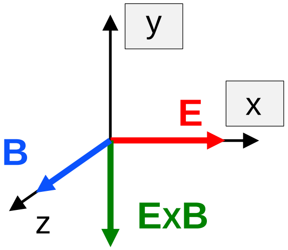
- **Test Case 2a - 2D axial-azimuthal PIC simulation of the ExB EDI**
  - Same as test Case 1 + More realistic conditions: take naturally into account finite axial length, density gradients, axial magnetic field profile, generation of electron and ion pairs by ionization



- **Test Case 2b - 2D radial-azimuthal PIC simulation of the ExB EDI**
  - Same as test Case 1 + Understand role of sheaths and electron-wall interaction. Quantify electron heating in the direction // B (i.e. ⊥ to the walls) and its role on electron-wall interaction.

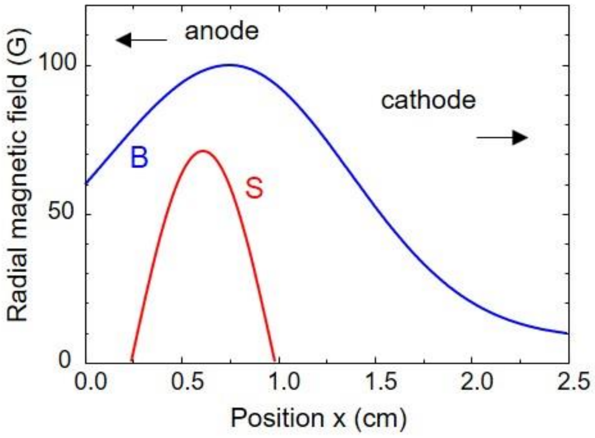


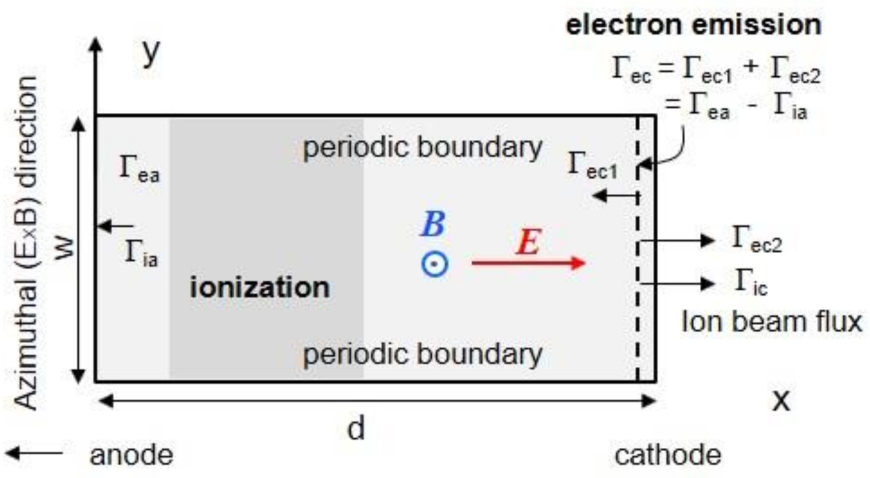
LANDMARK project – Some results



Test Case 2a - 2D axial-azimuthal PIC simulation of the EXB EDI

- 2D-3V axial-azimuthal PIC model
- Given B profile, applied voltage and ionization source term
- Total current density and plasma density adjusted by adjusting ionization source term
- Electron current entering the channel not imposed (must neutralize extracted ion beam).
- Periodic in azimuthal direction. 2.5 cm length in axial direction
- 1 cm in azimuthal direction.





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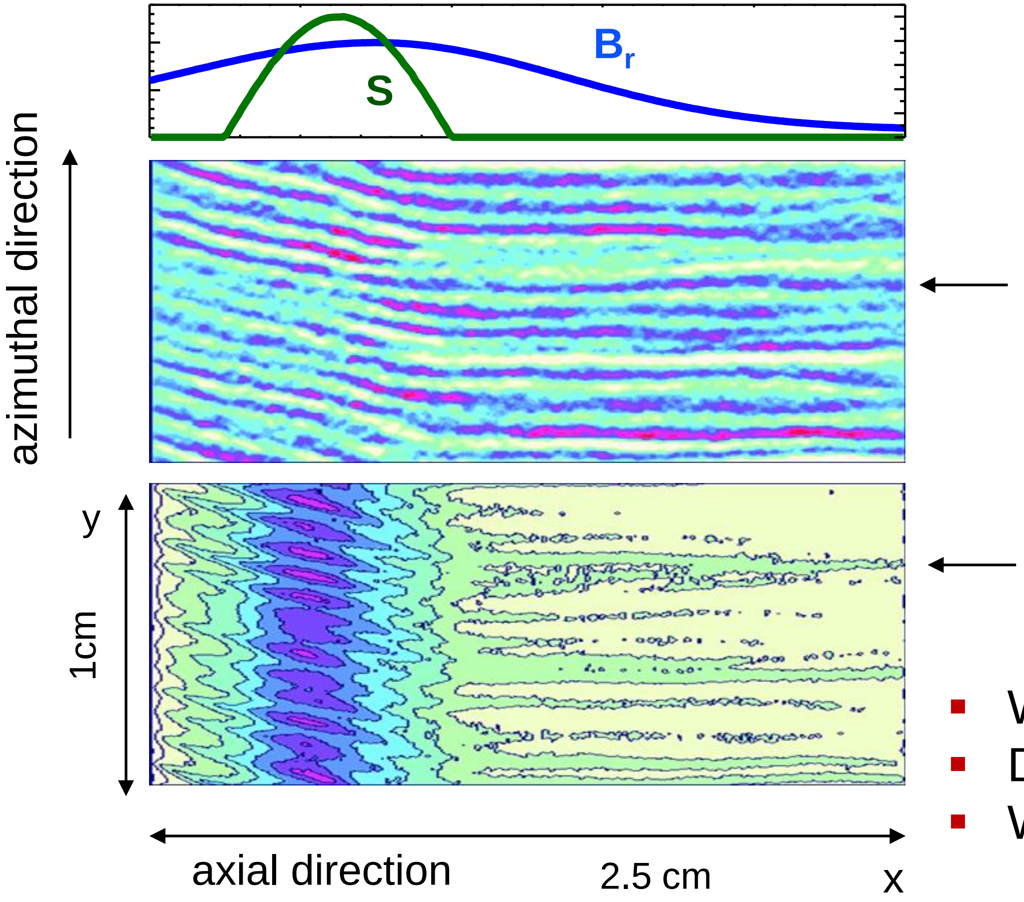
LANDMARK project – Some results

Test Case 2a - 2D axial-azimuthal PIC simulation of the ExB EDI

$$J = e\int_0^d S(x)dx = 400\ \mathrm{A/m^2}$$



azimuthal field E_y [-5×10^4 , $+5\times10^4$] V/m

Positive ion density [0 – 5×10^{17}] m^{-3}

- Wavelength of the instability ~1 mm
- Debye length ~ 50-100 μm
- Wave frequency on the order of ion plasma freq.

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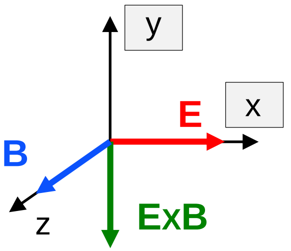
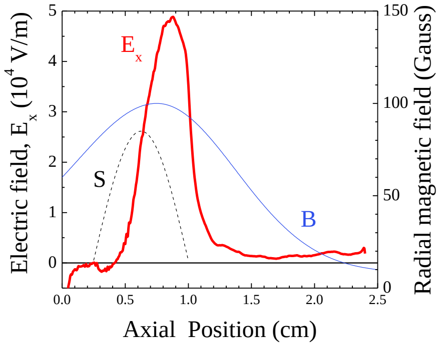
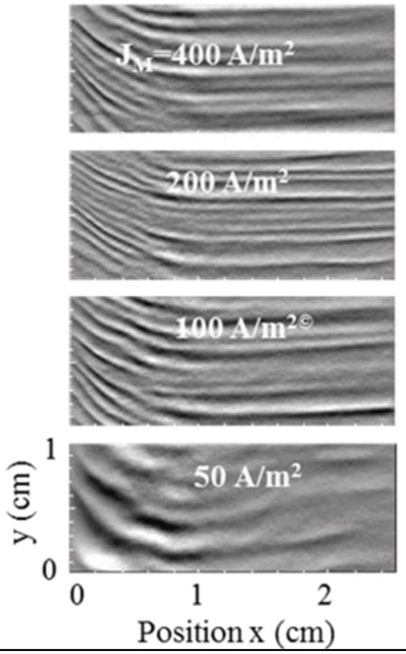
Test Case 2a - 2D axial-azimuthal PIC simulation of the EXB EDI



E_x



E_y



- Axial electric field (max ~5x10^4 V/m) distributed over 5-10 mm
- Azimuthal wave: Large azimuthal field (several 10^4 V/m)
- Wave length seems to scale as ~ λ_{De} (about 10 λ_{De})
- Wave frequency ~scales with ω_{pi}
- Amplitude of the azimuthal field decreases with decreasing plasma density; scales ~ as T_e/λ_{De}
- **Consistent with ion acoustic instability ?**

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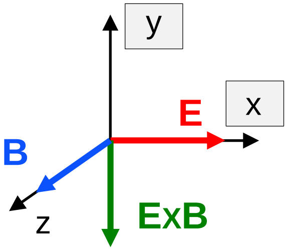
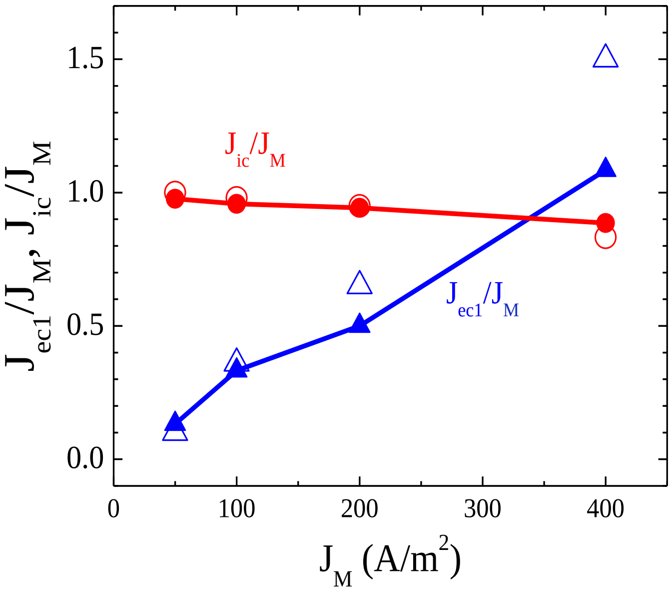


Test Case 2a - 2D axial-azimuthal PIC simulation of the EXB EDI

Electron current vs Total ion current



Effective Hall parameter, col. frequency, mobility

	h	ν_{eff} (10^6 s^{-1})	$\mu_{ex,eff}$ m^2/V/s
J_M=50 A/m^2	770	2.1	0.13
100 A/m^2	500	3.2	0.2
200 A/m^2	370	4.3	0.27
400 A/m^2	192	8.3	0.52

- Effective mobility at max B field more realistic than in 1D azimuthal or 2D radial-azimuthal simulations

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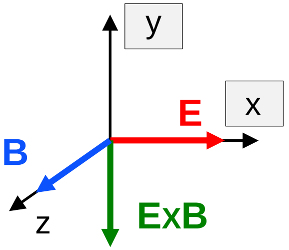
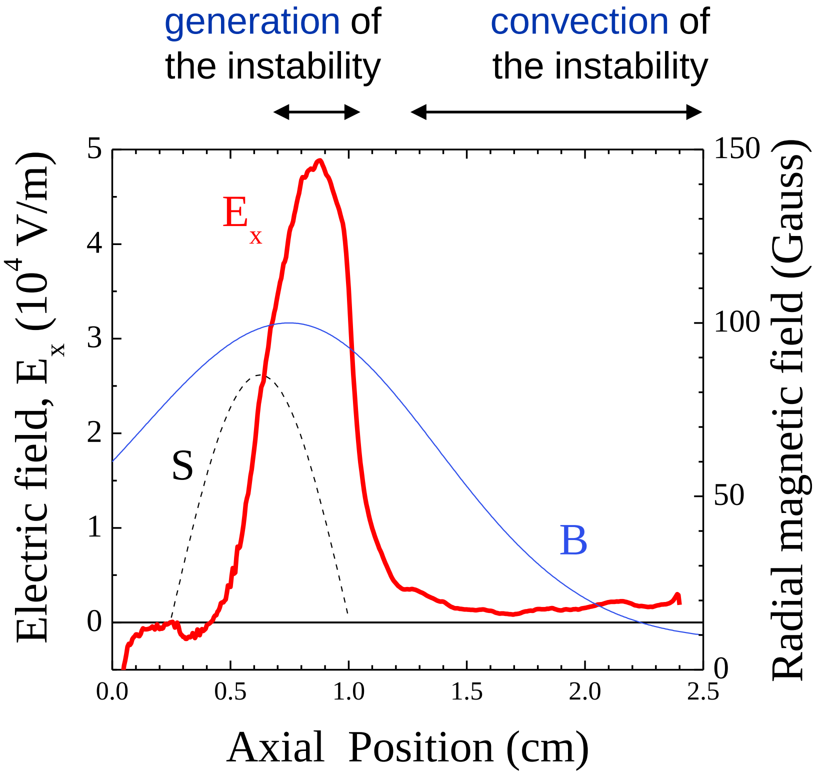



generation of the instability

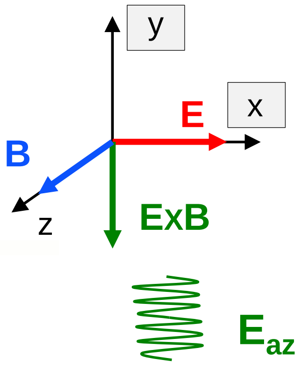
convection of the instability

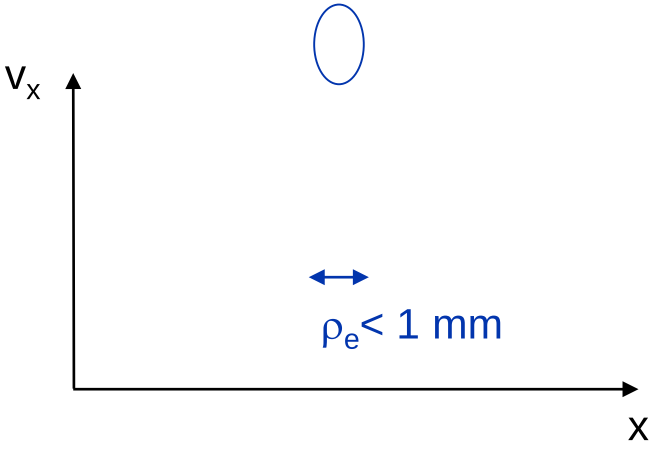
- The wave is generated in the acceleration region because of the large ExB drift
- The wave is convected downstream by ions. Electron-wave coupling is not important downstream of the acceleration region (low E/B)
- *Can we understand electron transport in this region in terms of test particle trajectories ?*

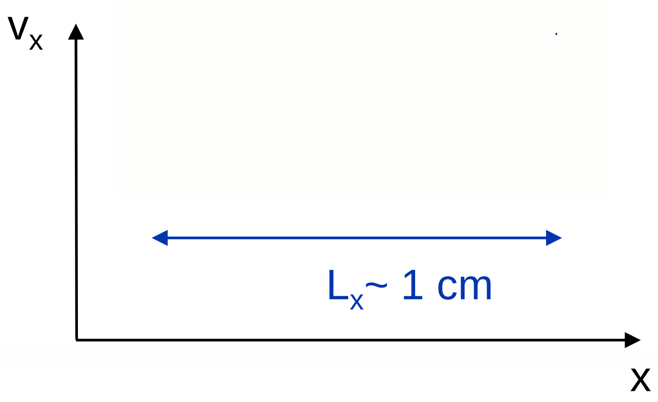
LANDMARK project – Some results

Test Case 2a - Test particle trajectories with azimuthal wave

E_x=1000 V/m, E_{az}=5x10^4 V/m, λ_{az}=1 mm, B=100 G,





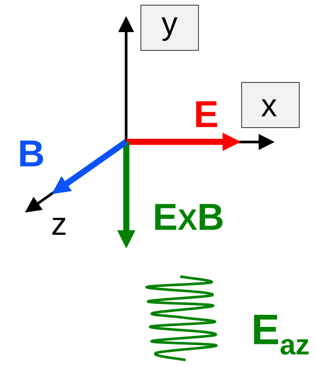


- When azimuthal field = 0 electron trajectory along x is bounded by cyclotron radius ρ_e

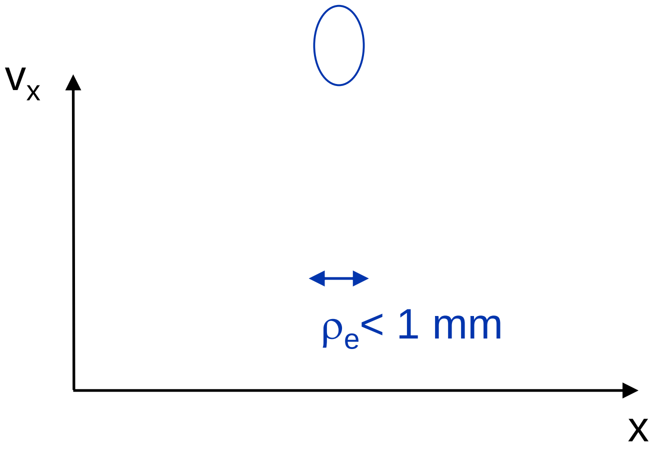
- When azimuthal field not zero trajectory along x may still be bounded but can be considerably elongated in the x direction.
- For given azimuthal field amplitude and wavelength, axial field, and magnetic field, elongation length L_x depends on initial electron velocity

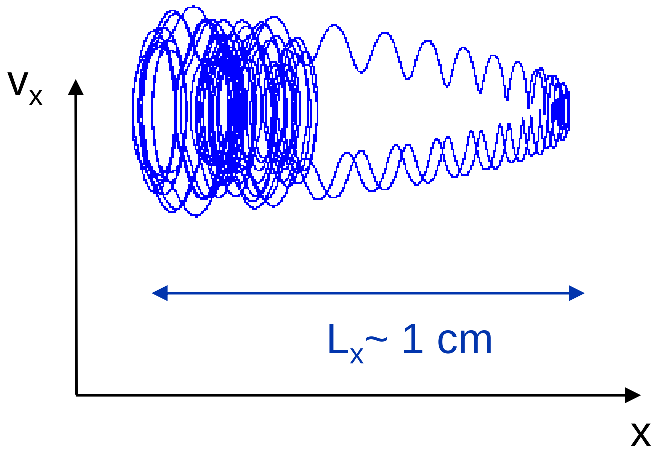
LANDMARK project – Some results

Test Case 2a - Test particle trajectories with azimuthal wave



E_x=1000 V/m, E_{az}=5x10^4 V/m, λ_{az}=1 mm, B=100 G,





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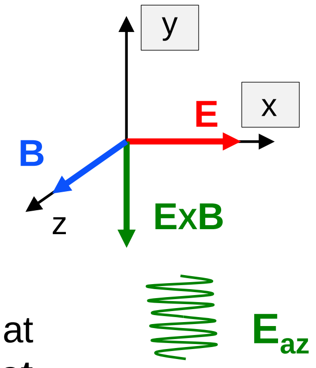
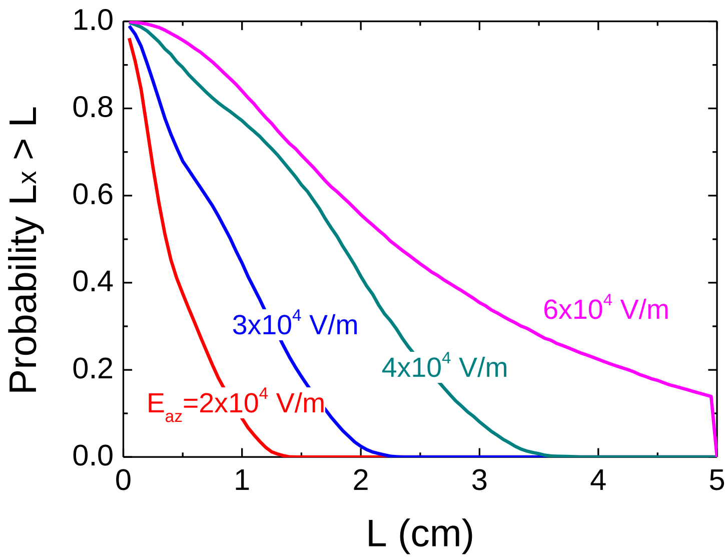
LANDMARK project – Some results

Test Case 2a - Test particle trajectories with azimuthal wave



E_x=1000 V/m, E_{az}=5x10^4 V/m, λ_{az}=1 mm, B=100 G,

- By choosing random initial velocity (e.g. according to a Maxwellian distribution at temperature T_e), and simulating electron trajectories, one can calculate the **probability that the electron trajectory is elongated by a length larger than L** in the anode direction



E_x=10^3 V/m
B=150 G
T_e=10 eV
λ_w=1 mm

- **Convected azimuthal wave is very efficient for cross-field electron transport in the region downstream of the acceleration region**

Conclusion

1. The ExB EDI is present in all PIC simulations where the EXB direction is included
2. The transition to ion acoustic may depend on conditions of the model
3. In the 2D axial-azimuthal case the wavelength seems to scale ~ as $1/\lambda_{De}$, the wave amplitude as T_e/λ_{De}, the wave frequency as ω_{pi}. Ion acoustic ?
4. Electron transport in the region downstream of the accceleration region, the convected instability seems to be sufficient to explain electron transport in the near plume region
5. Large values of azimuthal field amplitude realistic ? Long wavelengths not described by models .
6. **Experimental evidence ???**